Exercise 58

Find an equation of the tangent line to the curve $y = x^4 + 1$ that is parallel to the line 32x - y = 15.

Solution

Writing the given equation of the line as

$$y = 32x - 15,$$

we see that it has a slope of 32. The aim is to take the derivative of the given function and find where it's equal to 32.

$$y' = \frac{d}{dx}(x^4 + 1)$$
$$= \frac{d}{dx}(x^4) + \frac{d}{dx}(1)$$
$$= (4x^3) + (0)$$
$$= 4x^3$$

Set this equal to 32 and solve for x.

$$4x^3 = 32$$
$$x^3 = 8$$
$$x = 2$$

Plug this value of x into the given function to get the corresponding y-value on the curve.

$$y(2) = 2^4 + 1 = 17$$

Finally, determine the equation of the line with slope 32 that goes through the point (2, 17).

$$y - 17 = 32(x - 2)$$